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# New parameterization of the trinucleon wave function and its application to the $\pi$ $^3$He scattering length

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**Abstract.** We present a new parameterization of the trinucleon wave function. As a novel feature a separable parameterization for the complete wave function is given. In this way any calculation that considers two-body currents only is largely simplified. To demonstrate this we calculate the $\pi$ $^3$He scattering length in chiral-perturbation theory. We find reasonable agreement with experimental values inferred from data on level shifts in pionic $^3$He bound states. The relevance of the $\pi$-triton system for an alternative determination of the $\pi N$ scattering lengths is discussed.

**PACS.** 25.80.Ls Pion inclusive scattering and absorption – 25.80.Hp Pion-induced reactions – 25.10.+s Nuclear reactions involving few-nucleon systems – 21.45.+v Few-body systems

## 1 Introduction

In spite of the importance of three- and four-nucleon systems as a bridge between the deuteron and heavier nuclei, there is an appalling shortage of serious theoretical work done on meson-nuclear physics with these. One stumbling stone for calculations of meson-nuclear interactions by "intermediate-energy physicists" is the need for at least the trinucleon ($^3$He or $^3$H) Faddeev wave functions, which normally exist in the form of numerical tables computed by a different society, low-energy physicists. The necessity of a more accessible form for nonspecialists was realized a long time ago by Hajduk *et al.* [1], who presented the Faddeev amplitudes of the trinucleon wave function for different channels and particle permutations as separable analytical forms. The aim of this work is to improve the parameterization of ref. [1] in three respects:

- The parameterization is given for trinucleon wave functions derived from two modern nucleon-nucleon interactions, namely the CD Bonn [2] and the Paris potential [3].
- We give a separable expansion for the full antisymmetrized wave function. Previous works [1] parameterized the Faddeev components only.
- The trinucleon wave function is not separable in all partial waves over the full momentum range. Therefore we include an additional term in the parameterization that allows the inclusion of correlations. Especially for the pair wave functions where the $NN$ pair is in a $^3d_1$ state this turns out to be crucial.

In the early 1990s Weinberg argued, that as long as we restrict ourselves to interactions with pions only, three- and more-body interactions are suppressed. Thus, for those calculations, carried out either in chiral-perturbation theory or within a phenomenological approach, all that is needed is an appropriate parameterization of the nuclear wave function, which contains the momentum distribution of one active pair only. All remaining degrees of freedom can be integrated/summed separately. If this parameterization is given for the full antisymmetrized wave function, to do the actual calculation of a nuclear matrix element with a two-body operator will be just as complicated as the evaluation of the matrix element on the deuteron. In this paper we demonstrate that this program can be easily carried out for the example of the $\pi$ $^3$He scattering length.

The paper is organized in the following way: Some basic information about the trinucleon wave function and details of the procedure how the wave function is parameterized by means of simply analytical functions are given in the next section. In the third section the usefulness of the new parameterization is demonstrated by an explicit calculation of the $\pi$ $^3$He scattering length within chiral-perturbation theory. The paper ends with a short summary.

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## 2 Parameterization of the trinucleon wave function

The full antisymmetric wave function of a three-nucleon system, can be presented as a sum of three different Faddeev components, each corresponding to different particle permutations and projected on a particular set of partial waves [4]:

$$|\Psi\rangle = |\psi[(12)3]\rangle + |\psi[(23)1]\rangle + |\psi[(31)2]\rangle. \quad (1)$$

The individual components read for instance in coordinate space

$$\psi^\nu(r_{ij},\rho_k) = \langle r_{12}\rho_3\nu_{12}|\psi[(12)3]\rangle = \langle r_{23}\rho_1\nu_{23}|\psi[(23)1]\rangle = \langle r_{31}\rho_2\nu_{31}|\psi[(31)2]\rangle, \quad (2)$$

where $r$ and $\rho$ denote the pair and spectator coordinates, respectively, and the index $\nu$ denotes the relevant quantum numbers. In ref. [1] these Faddeev components were presented in separable form $v^\nu(r_{ij})w^\nu(\rho_k) = \psi^\nu(r_{ij},\rho_k)$.

A parameterization of the individual Faddeev components is still rather clumsy to use in situations where, *e.g.*, a two-body operator should act on a term with a "wrong" grouping, *i.e.* the operator and the pair wave function $v^\nu(r_{ij})$ do not involve the same particles. However, the action of a two-body operator on a completely antisymmetric state does not depend on the particle identifications in the operator. Therefore, we parameterize the total antisymmetrized wave function in terms of pair and spectator coordinates (or momenta) projected on different angular-momentum eigenstates. We choose the particle pair (12) as the active pair and define

$$\begin{aligned} v^\nu(r)w^\nu(\rho) = \langle r\,\rho\,\nu|\Psi\rangle &= \langle r\rho\nu|\psi[(12)3]\rangle \\ &+ \sum_{\nu_{23}} \langle r\rho\nu|r_{23}\rho_1\nu_{23}\rangle\langle r_{23}\rho_1\nu_{23}|\psi[(23)1]\rangle \\ &+ \sum_{\nu_{31}} \langle r\rho\nu|r_{31}\rho_2\nu_{31}\rangle\langle r_{31}\rho_2\nu_{31}|\psi[(31)2]\rangle\,. \end{aligned} \quad (3)$$

Here the $\langle r\rho\nu|r_{ij}\rho_k\nu_{ij}\rangle$ denote the necessary recoupling coefficients for angular momentum, spin and isospin and the $r_{ij}$ and $\rho_k$ are to be expressed in terms of $r$ and $\rho$ [4]. *E.g.*, one finds $\vec{r}_{23} = -\frac{1}{2}\vec{r} - \vec{\rho}$.

Physically one might expect that, if in a bound three-body system either the spectator or the pair is far off-shell, then it would be less likely to find also the other far off-shell. In other words, there should be some kind of correlation between the momenta or the corresponding coordinates, which is not present in the simple product. In short: there is no reason to expect the triton wave function to be separable. We shall allow for such a correlation by using a fit with a sum of two products. Over a large momentum range this seems to give sufficiently accurate results.

In order to get converged results for the trinucleon binding energy and the wave function, in principle, a fairly large number of three-nucleon partial waves is needed [5]. However, most of those partial waves give only a small contribution to the total wave function, cf., *e.g.*, ref. [6].

**Table 1.** Quantum numbers of the three-body channels. $s$, $\tau$, $l$, and $j$ refer to the spin, isospin, orbital and total angular momentum in the $NN$ subsystem and $L$ and $K$ are the relative orbital angular momentum of the spectator and the so-called channel spin [6].

| Channel no. | Label | Subsystem | $l$ | $s$ | $j^\pi$ | $\tau$ | $K$ | $L$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $^1s_0S$ | $^1s_0$ | 0 | 0 | $0^+$ | 1 | 1/2 | 0 |
| 2 | $^3s_1S$ | $^3s_1$ | 0 | 1 | $1^+$ | 0 | 1/2 | 0 |
| 3 | $^3s_1D$ | $^3s_1$ | 0 | 1 | $1^+$ | 0 | 3/2 | 2 |
| 4 | $^3d_1S$ | $^3d_1$ | 2 | 1 | $1^+$ | 0 | 1/2 | 0 |
| 5 | $^3d_1D$ | $^3d_1$ | 2 | 1 | $1^+$ | 0 | 3/2 | 2 |

Therefore, in the following treatment we shall restrict ourselves to the most important states, *i.e.* those where the $NN$ pairs are in the singlet spin state $^1s_0$ and in the triplet states $^3s_1$ or $^3d_1$. In this case the trinucleon wave function $\psi^\nu$ has only five components, whose quantum numbers are summarized in table 1. In line with this restriction we also include only those five components in the evaluation of the total wave function $|\Psi\rangle$ according to eq. (3). The higher partial waves induced by the antisymmetrization have much less weight. We should mention, however, that they still contribute to the overall normalization [7]. But, for convenience, the total wave functions employed in the present work are normalized in such a way that the sum over the five considered channels adds up to unity.

The procedure for the fit is the following. First the full Faddeev wave function (using the CD Bonn [2] and Paris potential [3]) is obtained numerically in momentum space, taking into account the permutation of the particles (cf. eq. (3)), and then projected on the 5 considered partial waves (channels). This gives the "exact" wave functions $\Psi^\nu(p,q)$ to be fitted in each of the 5 channels considered ($^1s_0S$, $^3s_1S$, $^3d_1S$, $^3s_1D$ and $^3d_1D$, cf. table 1) as functions of the momenta $p$ and $q$ for the pair and the spectator, respectively. Each of these is approximated by a product of functions $v_1^\nu(p)$ and $w_1^\nu(q)$ given by a five-term expansion of Lorentz functions,

$$v_\lambda^\nu(p) = \sum_{n=1}^{5} \frac{a_{n,\lambda}^\nu}{p^2 + (m_{n,\lambda}^\nu)^2}\,, \quad w_\lambda^\nu(q) = \sum_{n=1}^{5} \frac{b_{n,\lambda}^\nu}{q^2 + (M_{n,\lambda}^\nu)^2}\,. \quad (4)$$

Here for each $\nu$ and $\lambda$ the conditions

$$\sum_{n=1}^{5} a_{n,\lambda}^\nu = \sum_{n=1}^{5} b_{n,\lambda}^\nu = 0 \quad (5)$$

are required for convergence in momentum space and to guarantee regularity at the origin in coordinate space (for $D$-waves more conditions are needed [1,8]). We perform a standard $\chi^2$ fit to the total antisymmetrized exact (numerical) three-nucleon wave functions by minimizing the function

$$\int_0^\infty \mathrm{d}p\;\mathrm{d}q\; p^2q^2|\Psi^\nu(p,q) - v_1^\nu(p)w_1^\nu(q)|^2. \quad (6)$$

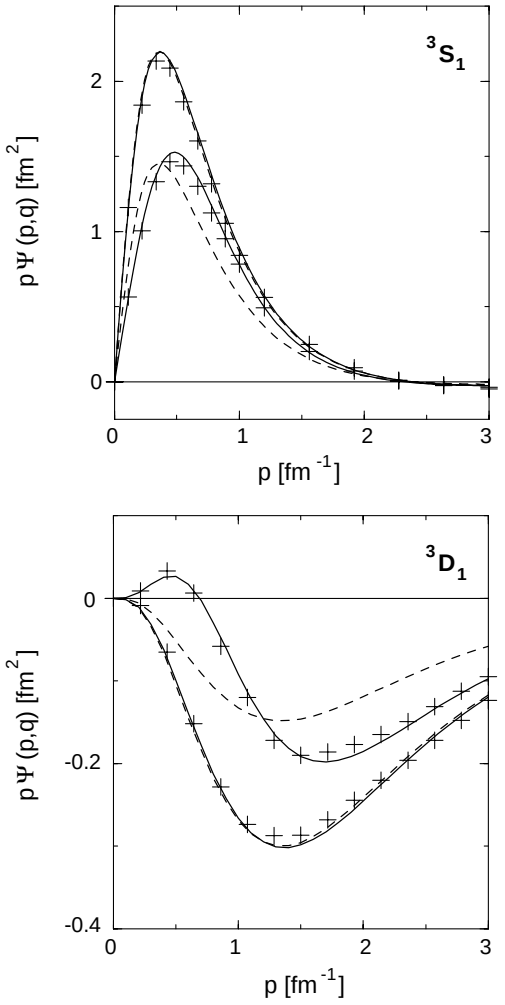


**Fig. 1.** The full antisymmetric wave function for the channels $^3s_1S$ (upper panel) and $^3d_1S$ (lower panel) as a function of the relative pair momentum $p$ for spectator momenta $q = 0.5$ (larger pair) and 1 fm$^{-1}$ (smaller pair) —the latter multiplied by a factor of three. For both momenta we show the single-term fit (dashed lines), the two-term fit (solid lines) and the exact wave function (+). Here we used the wave function derived from the CD Bonn potential.

Like Hajduk *et al.* [1] we include the additional weight factor $p^2q^2$, for this factor emphasizes more the relevant momentum range, as factors $p^2$ and $q^2$ always appear naturally in any matrix element as they originate from the integration measure. After the first fit, where only the term $v_1^\nu(p)w_1^\nu(q)$ is included, an additional product term $v_2^\nu(p)w_2^\nu(q)$ is fitted to the remaining deviation between this fit and the exact wave function. Thus we have

$$\Psi^\nu(p,q) = v_1^\nu(p)w_1^\nu(q) + v_2^\nu(p)w_2^\nu(q). \tag{7}$$

In this work we stop after the second term. However, it should be clear that every additional term included systematically improves the fit without influencing the previous terms.

We found that the inclusion of two terms is sufficient to reproduce the "exact" wave function well over a reasonably large momentum range. This is illustrated in fig. 1, where the one- and two-term fits are compared to the "exact" wave function in the $^3s_1S$ as well as the $^3d_1S$ channel for two different spectator momenta. In particular, for the $^3d_1S$ case the need for a second term is striking. Another way of testing the quality of the analytical parameterization is offered by the probability for the different components of the wave function in the Blatt-Derrick representation [6]. Corresponding results, for the CD Bonn as well as for the Paris potential, are compiled in table 2. Here the apparent nonseparability of the $^3d_1S$ is reflected in the $D$-state probability $P(D)$. With a single-term fit there is still a significant deviation of the value fitted for the CD Bonn potential (Paris potential) 6.995% (8.319%) from the exact one 7.127% (8.428%), while the two-term fit has essentially converged to the exact result with a significantly better value 7.082% (8.386%), cf. table 2.

**Table 2.** Probabilities (in the Blatt-Derrick representation) of the trinucleon wave function components for the CD Bonn and Paris potentials. "1 term" and "2 terms" refer to the single- and two-term parameterizations described in sect. 2.

| | $P(S)$ | $P(S')$ | $P(P)$ | $P(D)$ |
|---|---|---|---|---|
| CD Bonn 1 term | 91.77 | 1.187 | 0.047 | 6.995 |
| CD Bonn 2 terms | 91.60 | 1.274 | 0.048 | 7.082 |
| CD Bonn | 91.55 | 1.276 | 0.049 | 7.127 |
| Paris 1 term | 90.08 | 1.543 | 0.062 | 8.319 |
| Paris 2 terms | 89.85 | 1.660 | 0.100 | 8.386 |
| Paris | 89.90 | 1.610 | 0.066 | 8.428 |

In the coordinate space representation these approximate wave functions will consist of sums of Yukawa functions and (for $D$-waves) their derivatives

$$V_\lambda^\nu(r) = \sqrt{\frac{\pi}{2}} \sum_{n=1}^{5} a_{n,\lambda}^\nu \; e^{-m_{n,\lambda}^\nu r}$$

or

$$V_\lambda^\nu(r) = \sqrt{\frac{\pi}{2}} \sum_{n=1}^{5} a_{n,\lambda}^\nu \, e^{-m_{n,\lambda}^\nu r} \left(1 + \frac{3}{m_{n,\lambda}^\nu r} + \frac{3}{(m_{n,\lambda}^\nu r)^2}\right), \tag{8}$$

with similar expressions for the spectator $\rho$-dependence. As usual, in coordinate space an additional factor of $r$ is included in the definition of the wave functions ($V$ is $r$ times the Fourier transform of $v$).

Table 3 contains the parameters for the fit to the wave functions using the CD Bonn potential. It may be noted that there is some freedom in distributing the strength between the pair functions $v_\lambda^\nu$ and the spectator $w_\lambda^\nu$. This freedom is used to present the results in such a normalization that the square of the first term pair function integrates to unity, *i.e.* $\int_0^\infty \mathrm{d}p \; p^2 (v_1^\nu)^2 = 1$ for each partial wave $\nu$. Table 4 gives the parameters of the triton wave

**Table 3.** The parameters $a_n$, $b_n$, $m_n$ and $M_n$ of the fit for the full antisymmetric three-body wave function with the CD Bonn potential.

| State/term | $a_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $m_n$ ($\mathrm{fm}^{-1}$) | $b_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $M_n$ ($\mathrm{fm}^{-1}$) |
|---|---|---|---|---|
| $^1s_0S$ | −1.420402 | 0.408491 | −3.269355 | 0.587360 |
| Term 1 | −0.713402 | 0.660926 | 11.674343 | 0.787360 |
| | −1.500957 | 0.913361 | −33.930463 | 0.987360 |
| | 10.077357 | 1.165797 | 44.864561 | 1.187360 |
| | −6.442596 | 1.418232 | −19.339085 | 1.387360 |
| Term 2 | 0.621726 | 0.346344 | 0.230330 | 0.384846 |
| | 8.073312 | 0.546344 | 5.555461 | 0.735757 |
| | −39.510606 | 0.746344 | −27.949487 | 1.086668 |
| | 52.419034 | 0.946344 | 38.650601 | 1.437579 |
| | −21.603467 | 1.146344 | −16.486905 | 1.788490 |
| $^3s_1S$ | 1.308614 | 0.424997 | −4.354797 | 0.534346 |
| Term 1 | 1.057018 | 0.818066 | 17.513718 | 0.734346 |
| | −2.491967 | 1.211135 | −42.954883 | 0.934346 |
| | −3.044193 | 1.604204 | 49.777813 | 1.134346 |
| | 3.170527 | 1.997273 | −19.981851 | 1.334346 |
| Term 2 | 0.668573 | 0.359204 | −0.141018 | 0.319402 |
| | 8.171864 | 0.559204 | −2.573848 | 0.702473 |
| | −39.501304 | 0.759204 | 14.710142 | 1.085544 |
| | 51.788425 | 0.959204 | −21.510088 | 1.468615 |
| | −21.127558 | 1.159204 | 9.514812 | 1.851686 |
| $^3d_1S$ | 0.439080 | 0.550025 | −0.260857 | 0.431744 |
| Term 1 | 2.063638 | 1.214291 | 0.781535 | 0.705959 |
| | −26.827630 | 1.878557 | −4.200276 | 0.980175 |
| | 41.548562 | 2.542822 | 7.620099 | 1.254391 |
| | −17.223651 | 3.207088 | −3.940502 | 1.528606 |
| Term 2 | −0.068267 | 0.556661 | 0.458816 | 0.307065 |
| | 0.921148 | 1.096208 | 18.886372 | 0.828691 |
| | −2.720463 | 1.635755 | −114.616019 | 1.350317 |
| | 2.869650 | 2.175302 | 176.411566 | 1.871943 |
| | −1.002067 | 2.714849 | −81.140735 | 2.393569 |
| $^3s_1D$ | 7.856844 | 1.077420 | 0.008223 | 0.316062 |
| Term 1 | 16.344288 | 1.277420 | 0.376375 | 0.820171 |
| | −27.636271 | 1.477420 | −2.558819 | 1.324281 |
| | −26.578179 | 1.677420 | 3.621148 | 1.828390 |
| | 30.013317 | 1.877420 | −1.446927 | 2.332499 |
| Term 2 | −1.089313 | 0.666635 | 28.192605 | 0.828086 |
| | 9.975483 | 0.866635 | −226.000462 | 1.203061 |
| | −26.331496 | 1.066635 | 551.617505 | 1.578036 |
| | 27.163776 | 1.266635 | −530.511234 | 1.953012 |
| | −9.718449 | 1.466635 | 176.701587 | 2.327987 |
| $^3d_1D$ | −0.430426 | 0.345360 | 0.000745 | 0.180605 |
| Term 1 | 14.525808 | 0.921071 | −0.016356 | 1.497104 |
| | −59.378637 | 1.496783 | −0.340857 | 2.813602 |
| | 72.417582 | 2.072494 | 0.621826 | 4.130100 |
| | −27.134327 | 2.648206 | −0.265358 | 5.446598 |
| Term 2 | 0.017189 | 0.382127 | 1.692045 | 0.222982 |
| | −0.110339 | 0.582127 | −33.002368 | 0.422982 |
| | 0.209799 | 0.782127 | 119.183936 | 0.622982 |
| | −0.160679 | 0.982127 | −139.556966 | 0.822982 |
| | 0.044030 | 1.182127 | 51.683353 | 1.022982 |

**Table 4.** The parameters $a_n$, $b_n$, $m_n$ and $M_n$ of the fit for the full antisymmetric three-body wave function using the Paris potential.

| State/term | $a_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $m_n$ ($\mathrm{fm}^{-1}$) | $b_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $M_n$ ($\mathrm{fm}^{-1}$) |
|---|---|---|---|---|
| $^1s_0S$ | 1.185873 | 0.366138 | 2.492877 | 0.537309 |
| Term 1 | 1.764563 | 0.647808 | −8.176120 | 0.737309 |
| | −3.192238 | 0.929479 | 26.193477 | 0.937309 |
| | −3.305482 | 1.211149 | −37.021445 | 1.137309 |
| | 3.547284 | 1.492820 | 16.511211 | 1.337309 |
| Term 2 | 0.071144 | 0.246611 | 0.130706 | 0.356517 |
| | 10.443707 | 0.455327 | 3.775052 | 0.687907 |
| | −45.143466 | 0.664043 | −18.874600 | 1.019297 |
| | 58.923642 | 0.872758 | 26.307399 | 1.350687 |
| | −24.295028 | 1.081474 | −11.338557 | 1.682077 |
| $^3s_1S$ | 1.129463 | 0.382407 | −3.468556 | 0.482186 |
| Term 1 | 1.063093 | 0.803847 | 13.568797 | 0.682186 |
| | −1.858398 | 1.225288 | −34.549146 | 0.882186 |
| | −4.412162 | 1.646729 | 41.474326 | 1.082186 |
| | 4.078003 | 2.068170 | −17.025421 | 1.282186 |
| Term 2 | 0.536411 | 0.308809 | −0.084904 | 0.285273 |
| | 9.613061 | 0.508809 | −1.853015 | 0.653089 |
| | −45.528006 | 0.708809 | 10.592513 | 1.020905 |
| | 60.473587 | 0.908809 | −15.691503 | 1.388720 |
| | −25.095054 | 1.108809 | 7.036910 | 1.756536 |
| $^3d_1S$ | 0.400574 | 0.519557 | −0.238406 | 0.392032 |
| Term 1 | −0.136721 | 1.295772 | 0.469922 | 0.693166 |
| | −18.270490 | 2.071987 | −3.110884 | 0.994299 |
| | 31.444828 | 2.848202 | 6.387897 | 1.295433 |
| | −13.438191 | 3.624417 | −3.508528 | 1.596567 |
| Term 2 | −0.050950 | 0.504112 | 0.727812 | 0.331487 |
| | 0.788400 | 1.071790 | 17.453956 | 0.846965 |
| | −2.417995 | 1.639468 | −115.129833 | 1.362444 |
| | 2.592890 | 2.207146 | 182.257098 | 1.877922 |
| | −0.912345 | 2.774824 | −85.309033 | 2.393400 |
| $^3s_1D$ | −0.328513 | 0.432149 | 0.009219 | 0.297932 |
| Term 1 | 4.302698 | 0.813996 | 0.321081 | 0.804086 |
| | 7.984795 | 1.195843 | −2.366748 | 1.310240 |
| | −29.173703 | 1.577690 | 3.403634 | 1.816394 |
| | 17.214723 | 1.959537 | −1.367186 | 2.322548 |
| Term 2 | −0.028375 | 0.477081 | 53.452223 | 0.722757 |
| | 0.521663 | 0.777081 | −512.515615 | 1.152683 |
| | −1.805123 | 1.077081 | 1359.198606 | 1.582609 |
| | 2.190226 | 1.377081 | −1367.181577 | 2.012535 |
| | −0.878391 | 1.677081 | 467.046364 | 2.442462 |
| $^3d_1D$ | 0.053659 | 0.204481 | 0.000161 | 0.273758 |
| Term 1 | −6.461875 | 0.882230 | −0.187203 | 1.240855 |
| | 33.853359 | 1.559978 | 1.208634 | 2.207951 |
| | −44.645453 | 2.237726 | −1.685015 | 3.175047 |
| | 17.200310 | 2.915475 | 0.663424 | 4.142144 |
| Term 2 | −0.065878 | 0.165941 | 0.300000 | 0.638723 |
| | 4.577657 | 0.797478 | −2.649250 | 1.038723 |
| | −17.610419 | 1.429016 | 6.528825 | 1.438723 |
| | 20.615853 | 2.060553 | −6.235132 | 1.838723 |
| | −7.517212 | 2.692091 | 2.055558 | 2.238723 |

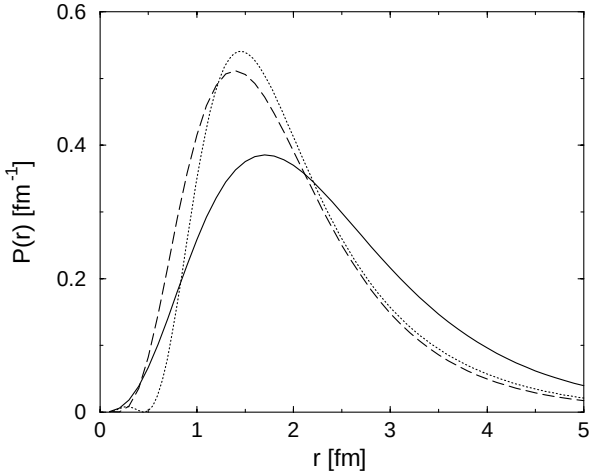


**Fig. 2.** The coordinate space probability distributions for the channel $^3s_1S$ with the spectator motion integrated out. Dotted curve: the representation of the Faddeev amplitude of Hajduk *et al.* [1]; dashed line: the same quantity, however, showing our fit to the wave function derived from the Bonn CD potential (cf. table 5); solid line: full antisymmetrized wave function.

function fit for the Paris potential. The parameters are available from the authors by e-mail[1].

As mentioned before, the parametrized wave functions $\Psi^\nu(p,q)$ are normalized in the usual way, *i.e.*

$$\sum_\nu \int_0^\infty \mathrm{d}p\, \mathrm{d}q\, p^2 q^2 |\Psi^\nu(p,q)|^2 = 1, \tag{9}$$

for the sum over the five channels considered. However, for completeness reasons we would like to point out that in a fully converged antisymmetrization of the wave function, with sufficient higher partial wave included, those five channels would have a relative weight of 0.954 (CD Bonn) and 0.945 (Paris), respectively. We would like to stress that the remaining 5% are saturated by a large number of partial waves that are individually rather small (cf. ref. [6]) and thus should be of minor relevance for the calculation of observables.

Figure 2 shows the probability distributions (pair correlations)

$$P(r) \equiv \int_0^\infty |\Psi^{(^3s_1S)}(r,\rho)|^2 \mathrm{d}\rho \tag{10}$$

integrated over the spectator degrees of freedom. For definiteness, these are all normalized to unity. Here, for comparison with the earlier work [1] (dotted line), we also include the fits to the individual Faddeev amplitude (dashed line). As one might expect, these results are rather similar to each other, while the antisymmetrized wave function gives a significantly longer-ranged distribution (solid line). It is interesting that the short-range node in the wave functions of ref. [1] is also present in our single-channel fits but

[1] Either from J. Niskanen (`Jouni.Niskanen@helsinki.fi`) or from J. Haidenbauer (`j.haidenbauer@fz-juelich.de`).

**Table 5.** The parameters $a_n$, $b_n$, $m_n$ and $M_n$ of the fit for single-permutation Faddeev amplitudes for the CD Bonn potential. Here the single-permutation normalization integral defined in ref. [1] is $N^2 = \langle\psi(1)|\psi(1)\rangle = 0.1597$.

| State/term | $a_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $m_n$ ($\mathrm{fm}^{-1}$) | $b_n$ ($\mathrm{fm}^{-\frac{1}{2}}$) | $M_n$ ($\mathrm{fm}^{-1}$) |
|---|---|---|---|---|
| $^1s_0S$ | −1.688022 | 0.532323 | −0.455000 | 0.499291 |
| Term 1 | 3.387087 | 0.945460 | −0.018469 | 0.699291 |
| | −19.727502 | 1.358596 | −0.116043 | 0.899291 |
| | 38.728212 | 1.771733 | 1.839667 | 1.099291 |
| | −20.699775 | 2.184870 | −1.250155 | 1.299291 |
| Term 2 | 0.085356 | 0.265782 | −0.000139 | 0.107800 |
| | 7.248584 | 0.676027 | 0.331163 | 0.461937 |
| | −38.887127 | 1.086272 | −1.640077 | 0.816075 |
| | 58.934705 | 1.496517 | 2.272345 | 1.170212 |
| | −27.381518 | 1.906762 | −0.963293 | 1.524350 |
| $^3s_1S$ | −1.426869 | 0.510733 | 0.003479 | 0.175113 |
| Term 1 | 0.526004 | 0.973756 | 0.819387 | 0.436154 |
| | −6.707522 | 1.436779 | −0.201750 | 0.697195 |
| | 18.647993 | 1.899802 | −1.766824 | 0.958236 |
| | −11.039606 | 2.362825 | 1.145708 | 1.219277 |
| Term 2 | 0.563686 | 0.311171 | −0.111000 | 0.341604 |
| | 5.367244 | 0.753186 | −0.665672 | 0.567744 |
| | −39.146731 | 1.195201 | 4.131617 | 0.793884 |
| | 63.040222 | 1.637216 | −5.800799 | 1.020025 |
| | −29.824421 | 2.079230 | 2.445854 | 1.246165 |
| $^3d_1S$ | 0.278338 | 0.476215 | −0.191666 | 0.416574 |
| Term 1 | 1.922824 | 1.189219 | 0.132050 | 0.667231 |
| | −23.998840 | 1.902224 | −1.267253 | 0.917888 |
| | 37.142550 | 2.615229 | 2.720458 | 1.168546 |
| | −15.344872 | 3.328234 | −1.393589 | 1.419203 |
| Term 2 | −0.008422 | 0.434522 | 2.864122 | 0.388367 |
| | 0.192188 | 1.123081 | 16.511009 | 0.717339 |
| | −0.653916 | 1.811640 | −116.582783 | 1.046312 |
| | 0.733997 | 2.500199 | 174.361137 | 1.375285 |
| | −0.263848 | 3.188758 | −77.153484 | 1.704258 |
| $^3s_1D$ | 0.761443 | 0.575896 | 0.020358 | 0.389875 |
| Term 1 | 1.462244 | 0.775896 | −0.171640 | 0.897031 |
| | −0.354613 | 0.975896 | 0.119029 | 1.404187 |
| | 1.482841 | 1.175896 | 0.131303 | 1.911344 |
| | −3.351914 | 1.375896 | −0.099050 | 2.418500 |
| Term 2 | −0.010448 | 0.610872 | 3.527742 | 0.888433 |
| | −0.194635 | 1.021958 | 29.340847 | 1.188433 |
| | 1.259114 | 1.433044 | −155.301234 | 1.488433 |
| | −2.105286 | 1.844130 | 201.332436 | 1.788433 |
| | 1.051255 | 2.255216 | −78.899791 | 2.088433 |
| $^3d_1D$ | 0.392036 | 0.756905 | 0.004056 | 0.394786 |
| Term 1 | 9.293162 | 1.399407 | 0.094926 | 0.825067 |
| | −54.630929 | 2.041908 | −0.620590 | 1.255347 |
| | 74.598555 | 2.684410 | 0.869670 | 1.685627 |
| | −29.652824 | 3.326911 | −0.348062 | 2.115907 |
| Term 2 | 0.018926 | 0.950962 | 34.649504 | 0.697745 |
| | −0.178877 | 1.526973 | −323.198246 | 1.098823 |
| | 0.468087 | 2.102984 | 846.220442 | 1.499901 |
| | −0.466509 | 2.678995 | −845.312868 | 1.900980 |
| | 0.158373 | 3.255006 | 287.641168 | 2.302058 |

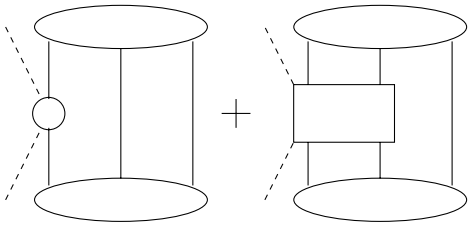

**Fig. 3.** Leading-order contributions to the $\pi\,^3$He scattering length.

not in the full antisymmetric wave function. For completeness, table 5 gives the parameterization fully analogous to ref. [1] used to produce the dashed curve. With the exception of the dotted line, the results in this figure are based on the CD Bonn potential.

## 3 The $\pi\,^3$He scattering length

Now we want to demonstrate the usefulness of the new parameterization by an explicit calculation of the real part of the $\pi\,^3$He scattering length in the chiral-perturbation theory. Since all the complications of having to treat a three-nucleon problem were already solved on the level of deriving the wave function parameterization in terms of an active pair and a spectator (cf. eq. (3)), the remaining part of the calculation for the scattering length is as complicated as the corresponding calculation for the two-nucleon system (cf., *e.g.*, refs. [9–11], where the $\pi d$ scattering length was calculated), as long as only two-nucleon operators are included as is the case for the NLO calculation we are going to perform here. Note, however, that with the parameterization given, also the explicit evaluation of three-body forces is largely simplified. In addition, the very simple form of the terms in the separable expansion allows an analytical calculation —the final result can be expressed in terms of incomplete $\Gamma$-functions.

We will work with the so-called hybrid approach, introduced by Weinberg [12], namely the $\pi NNN \to \pi NNN$ transition operators, evaluated using chiral-perturbation theory, will be convoluted with phenomenological wave functions using the parameterizations given in the previous section. It should be stressed that, recently, three-body wave functions were derived utilizing $NN$ and $3N$ forces that are consistent with the chiral counting scheme [13]. As will become clear below, once a parameterization in the form presented in this paper for those wave functions is available as well, it is a simple task to extract the corresponding $\pi\,^3$He scattering length.

Within the counting scheme advocated by Weinberg all that contributes to $\pi\,^3$He scattering at leading and next-to-leading order are one- and two-body currents, as illustrated in fig. 3. Due to the Goldstone nature of the pions, all three-body currents are suppressed by an additional factor $p^2/\Lambda^2$, where $p$ denotes the typical momentum of the problem and $\Lambda$ denotes the chiral-symmetry breaking scale ($\simeq 1$ GeV) [12].

In order to have a theory with predictive power it is compulsory that unknown counterterms are either suppressed or at least small in number. Naturally, in different schemes those terms appear at different places. This is not only true for the three-body system itself, where in a pion-less approach three-body forces appear at leading order [14], but also for the scattering process. For example, in ref. [15] it was stressed that in an approach using perturbative pions there appear counterterms in the production operator already at leading order in a calculation for $\pi d$ scattering. However, in this paper we will not elaborate on those subtleties any further but will use the approach of ref. [12].

It is a straightforward task to relate the one-body contribution to the scattering length. One gets

$$a_{\rm He}^{\rm (1b)} = \kappa \left( A a^{(+)} - Q_\pi \, a^{(-)} 2 T_3 \right) \,, \tag{11}$$

where $\kappa = \left(1 + \frac{m_\pi}{m_N}\right) / \left(1 + \frac{m_\pi}{A m_N}\right)$. Here $a^{(+)}$ ($a^{(-)}$) denote the isoscalar (isovector) $s$-wave $\pi N$ scattering length, $A$ is the number of nucleons in the nucleus (here $A = 3$), $Q_\pi$ is the charge of the pion in units of $e$ and $T_3$ denotes the third component of the isospin of the nucleus. Since the interaction is momentum independent, the loops occurring are identical to the expression of the wave function normalization. In contrast to the case of pion scattering on the deuteron, here the isovector piece of the scattering length does contribute and dominates the one-body contribution. We thus get for the one-body contribution of the scattering of negatively charged pions on $^3$He

$$a_{\rm He}^{\rm (1b)} = (92 \pm 15) \times 10^{-3} \; m_\pi^{-1} \,, \tag{12}$$

where we used $a^{(-)} = (90.5 \pm 4.2) \times 10^{-3} \; m_\pi^{-1}$ and $a^{(+)} = (-2.2 \pm 4.3) \times 10^{-3} \; m_\pi^{-1}$. These are the purely hadronic values presented in ref. [16], where Coulomb effects as well as isospin breaking effects are already subtracted. Note, that a consistent next–to–leading-order calculation would require to use $\pi N$ scattering lengths extracted from a calculation for $\pi N$ scattering carried out to the corresponding order as given, *e.g.*, in ref. [17]. In addition, a careful analysis would also necessitate to take into account isospin breaking effects consistently, as was stressed in ref. [18]. One expects, however, that especially the latter point enlarges primarily the theoretical error. Anyway, here we will ignore such subtleties because we only want to investigate, more qualitatively, if the $\pi\,^3$He system allows to extract information on the $\pi N$ scattering lengths.

Now we need to evaluate the matrix element of the wave function with the two-body scattering kernel $\mathcal{A}(\vec{p}\,', \vec{p})$, where $\vec{p}$ ($\vec{p}\,'$) denote the relative momentum of the active nucleon pair in the initial (final) state, respectively. Thus, we can write

$$a_{\rm He}^{\rm (2b)} = \frac{1}{4\pi} \binom{A}{2} \left( \frac{1}{1 + \frac{m_\pi}{A m_N}} \right) \times \int \mathrm{d}^3p' \, \mathrm{d}^3p \, \mathrm{d}^3q \, \Psi(\vec{p}\,', \vec{q})^\dagger \mathcal{A}(\vec{p}\,', \vec{p}) \Psi(\vec{p}, \vec{q}) \,. \tag{13}$$

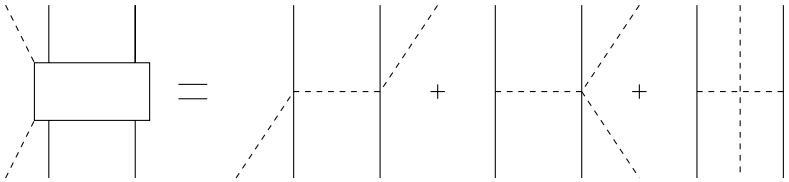

**Fig. 4.** Leading two-body contributions to the $\pi\,^3$He scattering length.

Here we already used the property that the wave function is antisymmetrized. Thus, whichever of the three nucleons is the spectator, the result of the matrix element is the same. Therefore, the inclusion of all possible pairwise interactions just leads to an overall factor of 3. This factor can be easily generalized to the case of a nucleus composed of $A$ nucleons. Then, there is a combinatorial factor which is given by the binomial coefficient $\binom{A}{2}$. If we now use the separable form presented in the previous section, we get

$$a_{\rm He}^{\rm (2b)} = \frac{1}{4\pi}\binom{A}{2}\left(\frac{1}{1+\frac{m_\pi}{Am_N}}\right)\sum_{\nu\nu'}\sum_{\lambda'\lambda}\mathcal{W}^{\nu'\nu}_{\lambda'\lambda} \times \int \mathrm{d}^3p'\,\mathrm{d}^3p\, v(p')^{\nu'}_{\lambda'}\langle\nu',\hat{p}\,'|\mathcal{A}(\vec{p}\,',\vec{p})|\nu,\hat{p}\rangle v(p)^{\nu}_{\lambda}\,, \tag{14}$$

where

$$\mathcal{W}^{\nu'\nu}_{\lambda'\lambda} = \int \mathrm{d}^3q\, w(q)^{\nu'}_{\lambda'} w(q)^{\nu}_{\lambda}\langle\nu',\hat{q}|\nu,\hat{q}\rangle\ . \tag{15}$$

Here $\lambda$ and $\lambda'$ denote the terms of the separable ansatz, $\nu'$ as well as $\nu$ denote the partial waves for the final and initial state, respectively, and the different state vectors $|\nu,\hat{p}\rangle$ contain the relevant spin, isospin and angular-momentum components of the corresponding partial wave.

For the two-body scattering kernel, as shown diagrammatically in fig. 4, we use [9]:

$$\mathcal{A}(\vec{p}\,',\vec{p}) = \frac{1}{(2\pi)^3}\frac{m_\pi^2}{4F_\pi^4}\left\{\frac{1}{\vec{q}^{\,2}}\left[2\delta^{\rm (ac)}(\vec{\tau}^{\,(1)}\cdot\vec{\tau}^{\,(2)}) - \tau^{(1)\rm a}\tau^{(2)\rm c} - \tau^{(1)\rm c}\tau^{(2)\rm a}\right] - g_A^2\frac{(\vec{\sigma}^{\,(1)}\cdot\vec{q})(\vec{\sigma}^{\,(2)}\cdot\vec{q})}{(\vec{q}^{\,2}+m_\pi^2)^2} \times\left[\delta^{\rm (ac)}(\vec{\tau}^{\,(1)}\cdot\vec{\tau}^{\,(2)}) - \tau^{(1)\rm a}\tau^{(2)\rm c} - \tau^{(1)\rm c}\tau^{(2)\rm a}\right]\right\}, \tag{16}$$

where the superscripts a (c) denote the isospin of the incoming (outgoing) pion, and $\vec{q}=\vec{p}\,'-\vec{p}$ denotes the pion momentum. It should be stressed that the same kernel was used recently to extract the isoscalar $\pi N$ scattering length from $\pi d$ scattering data [9]. Thus —as long as we restrict ourselves to $S$-waves in the bound-state wave functions— all we need to evaluate are the following two integrals:

$$I_1(m_i,m_j) = \frac{1}{4\pi}\int \mathrm{d}^3p\,\mathrm{d}^3p'\,\frac{1}{\vec{p}^{\,2}+m_i^2} \times\frac{1}{(\vec{p}-\vec{p}\,')^2}\,\frac{1}{\vec{p}^{\,\prime\,2}+m_j^2}\,, \tag{17}$$

$$I_2(m_i,m_j) = \frac{1}{4\pi}\int \mathrm{d}^3p\,\mathrm{d}^3p'\,\frac{1}{\vec{p}^{\,2}+m_i^2} \times\frac{(\vec{p}-\vec{p}\,')^2}{((\vec{p}-\vec{p}\,')^2+m_\pi^2)^2}\,\frac{1}{\vec{p}^{\,\prime\,2}+m_j^2}\,. \tag{18}$$

Obviously, the inclusion of $D$-waves can be done easily. Since the formulas are rather lengthy, we do not give them here explicitly. It should be stressed that the contribution due to the $D$-waves does not exceed 10%, which is of the order of the theoretical uncertainty induced by both the convergence rate of the chiral expansion and the model dependence of the scattering length caused by the particular employed nuclear wave function. We find

$$I_1(m_i,m_j) = \pi^3\Gamma((m_i+m_j)\epsilon,0)\,, \tag{19}$$

$$I_2(m_i,m_j) = \pi^3\left\{\Gamma((m_i+m_j+m_\pi)\epsilon,0) -\frac{1}{2}\left(\frac{m_\pi}{m_i+m_j+m_\pi}\right)\right\}, \tag{20}$$

where we used the incomplete $\Gamma$-function defined through $\Gamma(\lambda,n)=\int_\lambda^\infty \mathrm{d}x\,x^{n-1}\exp(-x)$ [19]. The constant $\epsilon$ was introduced to render the integrals finite. Equation (5) implies that the results in eqs. (19) and (20) are independent of $\epsilon$ as long as $m_i\epsilon \ll 1$ for all $i$.

We thus get for the two-body contribution to the $\pi\,^3$He scattering length

$$a_{\rm He}^{\rm (2b)} = \binom{A}{2}\left(\frac{1}{1+\frac{m_\pi}{Am_N}}\right)\left(\frac{m_\pi^2}{32\pi^4F_\pi^4}\right) \times\sum_\nu\sum_{\lambda',\lambda=1}^{2}\mathcal{W}^{\nu\nu}_{\lambda'\lambda}\left(\frac{2}{3}T_\nu\,(T_\nu+1)-1\right) \times\sum_{m,n=1}^{5}a^\nu_{m,\lambda'}a^\nu_{n,\lambda}\left\{I_1(m^\nu_{m,\lambda'},m^\nu_{n,\lambda}) -\frac{g_A^2}{4}\left(\frac{2}{3}S_\nu\,(S_\nu+1)-1\right)I_2(m^\nu_{m,\lambda'},m^\nu_{n,\lambda})\right\}. \tag{21}$$

Again, only the formula for the $S$-wave part of the wave function is given explicitly, although the effect of the

**Table 6.** Results for the $\pi$ $^3$He scattering length. The first three entries contain the measured energy level shifts of $\pi$ $^3$He atomic bound states together with the corresponding scattering lengths extracted by using eq. (24). The averaged values includes an additional error of 10% to take into account possible effects from isospin breaking, etc., cf. text.

| | $\epsilon_{1s}$ (eV) | $a_{\rm He}$ $(m_\pi^{-1}) \times 10^3$ |
|---|---|---|
| R. Abela *et al.* [20] | $44 \pm 5$ | $56 \pm 6$ |
| G.R. Mason *et al.* [21] | $34 \pm 4$ | $43 \pm 5$ |
| I. Schwanner *et al.* [22] | $32 \pm 3$ | $41 \pm 4$ |
| Averaged value | | $47 \pm 8 \pm 5$ |
| Theoretical prediction | | $67 \pm 15$ |

$D$-waves is included in the final result. Under this restriction, the partial wave of the initial $NN$ pair is equal to that of the outgoing pair.

Using the parameters for the two different wave functions as listed in tables 3 and 4, we get for the two-body contributions to the real part of the scattering length

$$a_{\rm He}^{\rm (2b)} = \begin{cases} -24 \times 10^{-3}\ m_\pi^{-1}\ \text{(Paris)}\,, \\ -26 \times 10^{-3}\ m_\pi^{-1}\ \text{(CD Bonn)}\,. \end{cases} \tag{22}$$

Compared to the leading one-body term we thus find reasonable convergence. Also the dependence on the wave function used can be regarded as a higher-order effect. As the final result we therefore have

$$\mathrm{Re}(a_{\rm He}^{\rm (th)}) = (67 \pm 15) \times 10^{-3}\ m_\pi^{-1}\ . \tag{23}$$

The error included is only that stemming from the error on the $\pi N$ scattering lengths used (cf. eq. (12)). As mentioned before, for a reliable estimate of the theoretical error a consistent calculation including isospin breaking effects as well as using consistent wave functions would be necessary. One could regard the size of the NLO contribution as a rather conservative estimate for the theoretical error.

Information on the $\pi$ $^3$He scattering length can be inferred from experimental information on level shifts caused by the strong interaction in the bound-state energies of $\pi^-$ $^3$He atoms [20–22]. A list of results from different experiments available in the literature is given in table 6. From those energy shifts scattering lengths can be easily extracted using the so-called Deser formula [23],

$$\epsilon_{1s} = -2(Z\alpha)^3 \mu^2\, \mathrm{Re}\left(a_{\rm He}^{\rm (exp)}\right)\ . \tag{24}$$

where $Z$ is the charge number of the nucleus and $\mu$ is the reduced mass of the $\pi$ $^3$He system. Taking the arithmetic mean value for the experimental results given in table 6, we thus get

$$\mathrm{Re}(a_{\rm He}^{\rm (exp)}) = (47 \pm 13) \times 10^{-3}\ m_\pi^{-1}\ , \tag{25}$$

where the error includes both the spread in the experimental numbers as well as an additional 10% to account for the omission of electromagnetic and isospin breaking corrections in eq. (24). Note that in case of pionic hydrogen those corrections turned out to be of the order of 7% [24]. Thus, we find agreement between theory and experiment within the given uncertainties.

As was argued above, at the current stage we cannot give a reliable error estimate for the theoretical calculation. Naturally, further studies are needed to draw stronger conclusions. However, it should be stressed, that it would be very useful to push the calculation as well as the experiment for the $\pi$ $^3$He system to as high an accuracy as those for the $\pi d$ system, since remaining discrepancies between theory and experiment in such a combined analysis can only stem from three-body currents and would therefore be an important test of chiral-perturbation theory in few-body systems.

In this context let us also emphasize that an experimental study of pionic tritium would be of very high interest because a combined analysis of the $\pi$ $^3$He and the $\pi t$ systems promises to provide direct access to the pion-nucleon scattering lengths. From eqs. (11) and (21) we find

$$\begin{aligned} a_t + a_{\rm He} &= 6\kappa a^{(+)} + \mathcal{O}\left(\left(\frac{m_\pi}{M_N}\right)^3\right)\ , \\ a_t - a_{\rm He} &= 2\kappa Q_\pi\, a^{(-)} + \mathcal{O}\left(\left(\frac{m_\pi}{M_N}\right)^4\right)\ , \end{aligned} \tag{26}$$

where the constant $\kappa$ was defined in eq. (11). Note, that for the extraction of $a^{(-)}$ one should expect the corrections to be suppressed by one chiral order compared to that of $a^{(+)}$. This is the case, because the leading corrections, that in the counting advocated by Weinberg appear at $\mathcal{O}((m_\pi/M_N)^3)$ —as given in eq. (21) for only $s$-waves in the bound-state wave function— do not distinguish between the $t$ and $^3$He system and thus do not contribute to the difference of the corresponding scattering lengths. Indeed a study of $\pi^- t$ bound states appears experimentally feasible nowadays [25]. It should be stressed that eqs. (26) are general and independent of the wave functions used. The error estimate only requires that the Weinberg scheme is applicable to the reactions under investigation. In addition, it should be clear that the $^3$He/$t$ system is an ideal system to study isospin breaking effects in few-nucleon systems.

It turns out that for the $\pi$ $^3$He scattering the omission of the second term in the expansion of the wave function (cf. eq. (7)) leads to a change in the two-body contribution of the scattering length by 2% only. Thus, in the energy range relevant for $\pi$ scattering at threshold, the full wave function is well described by a one-term separable form. As indicated in fig. 1 in reactions with either large momentum transfer or where higher partial waves become relevant, the effect of the second term is expected to be much more pronounced. Indeed, this has been already seen in a recent application of the wave function parametrization to $\pi$ absorption on $^3$He [26].

It is illuminating to compare eq. (21) with that for the two-body contribution in the case of pion-deuteron scattering. Obviously, in this situation it does not make sense to use two different terms expanded individually (thus, the

summation on $\lambda$ and $\lambda'$ disappears). In addition, there is no third particle (thus $\mathcal{W}^{\nu\nu}_{\lambda'\lambda} = 1$). All the rest remains unchanged. If we now use for the deuteron wave function a parametrization that was provided in ref. [27] for the Bonn B $NN$ model we reproduce the result obtained in ref. [9].

In the present work we do not aim at a highly accurate determination of the $\pi\,^3$He scattering length, but rather at a demonstration of the usefulness of the new parameterization. However, it should be clear that it is straightforward to improve the calculation. Results to order $p^4$ for the $\pi d$ scattering length are given in ref. [18] and could be used as the basis for an equally accurate calculation for the $\pi\,^3$He scattering length.

Already in ref. [9] it was observed that the contribution from the first diagram on the right-hand side in fig. 4 is significantly larger than that from the other two, although all three formally appear at the same order in the Weinberg scheme. In ref. [18] this was traced back to the small momentum scale introduced by the small binding energy of the deuteron. To account for this a modified counting scheme was advocated, leading to a reordering of the meson exchange diagrams: the first diagram on the right-hand side of fig. 4 is now the leading two-nucleon current, whereas the other two are down by 4 orders in the expansion parameter (cf. table 4 in ref. [18]). It is interesting to note that in our calculation we find a similar hierarchy of the results for the diagrams shown in fig. 4: the result for the first diagram exceeds the sum of the latter two by a factor of 20 or more, depending on the wave function used. Note that, similarly to the case of the deuteron, the typical momentum of a nucleon within $^3$He is significantly smaller than the pion mass.

Also a calculation of the imaginary part of the scattering length is, in principle, feasible. However, the contributions to the imaginary part are connected to the pion absorption on $^3$He, where a different counting scheme needs to be applied to account for the large nucleon momenta in the purely nucleonic intermediate state [28]. However, within this scheme the imaginary part should be suppressed by a factor of $(m_\pi/M_N)^{(3/2)}$ compared to $\mathrm{Re}(a^{2\mathrm{b}})$ given in eq. (21). The same holds true for the so-called dispersive corrections, namely the contribution of the inelastic channels to the real part of the scattering length.

## 4 Summary

In this paper we have given a new parameterization for two different trinucleon wave functions based on modern $NN$ interaction models. Although the procedure is in spirit similar to an earlier work [1], it extends the single-term separability to two terms allowing some correlation between the two relative momenta (or coordinates) and also approximates directly the full antisymmetrized wave function projected onto different partial waves. The former feature might be expected to be useful in phenomena involving high momenta, whereas the latter drastically simplifies calculations involving two-nucleon operators only.

Apparently, similar benefits are expected in applications to other reactions. *E.g.*, a calculation of quasifree pion absorption on nucleon pairs in $^3$He based on the presented parameterization has been reported in ref. [26].

As a demonstration the parameterization has been applied to calculate the $\pi\,^3$He scattering length. It has been seen that for such a parameterization the two-body contribution becomes very easy to compute even analytically. The actual calculation done in NLO chiral-perturbation theory yields a value of $\mathrm{Re}(a_{\mathrm{He}}) = (67 \pm 15) \times 10^{-3}\ m_\pi^{-1}$, which is in reasonable agreement with experimental values inferred from data on level shifts in pionic $^3$He bound states. We have argued that a combined analysis of $\pi d$, $\pi\,^3$He and $\pi t$ scattering should provide both important information on the $\pi N$ $s$-wave scattering lengths as well as a test of the applicability of chiral-perturbation theory to few-body systems.

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